Biostatistics: A Refresher

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Purdue University College of Pharmacy
Indiana University School of Medicine
West Lafayette and Indianapolis, Indiana
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Learning Objectives

1. Describe differences between descriptive and inferential statistics.
2. Identify different types of data (nominal, ordinal, continuous [ratio and interval]) to determine an appropriate type of statistical test (parametric vs. nonparametric).
3. Describe strengths and limitations of different types of measures of central tendency (mean, median, and mode) and data spread (standard deviation, standard error of the mean, range, and interquartile range).
4. Describe the concepts of normal distribution and the associated parameters that describe the distribution.
5. State the types of decision errors that can occur when using statistical tests and the conditions under which they can occur.
6. Describe hypothesis testing, and state the meaning of and distinguish between p-values and confidence intervals.
7. Describe areas of misuse or misrepresentation that are associated with various statistical methods.
8. Select appropriate statistical tests on the basis of the sample distribution, data type, and study design.
9. Interpret statistical significance for results from commonly used statistical tests.
10. Describe the similarities and differences between statistical tests, and state how to apply them appropriately.
11. Identify the use of survival analysis and different ways to perform and report it.

Self-Assessment Questions

Answers and explanations to these questions may be found at the end of the chapter.

1. A randomized controlled trial assesses the effects of the treatment of heart failure on global functioning in three groups of adults after 6 months of treatment. Investigators wanted to assess global functioning with the New York Heart Association (NYHA) functional classification, an ordered scale from I to IV, and to compare the patient classification after 6 months of treatment. Which statistical test is most appropriate to assess differences in functional classification between the groups?

A. Kruskal-Wallis test.
B. Wilcoxon signed-rank test.
C. Analysis of variance (ANOVA).
D. Analysis of covariance (ANCOVA).

2. You are evaluating a randomized, double-blind, parallel-group controlled trial that compares four antihypertensive drugs for their effect on blood pressure. The authors conclude that hydrochlorothiazide is better than atenolol (p<0.05) and that enalapril is better than hydrochlorothiazide (p<0.01), but no difference is observed between any other drugs. The investigators used an unpaired (independent samples) t-test to test the hypothesis that each drug was equal to the other. Which statement is most appropriate?

A. Investigators used the appropriate statistical test to analyze their data.
B. Enalapril is the most effective of these drugs.
C. ANOVA would have been a more appropriate test.
D. A paired t-test is a more appropriate test.

3. In the results of a randomized, double-blind, controlled clinical trial, the difference in hospital readmission rates between the intervention group and the control group is 6% (p=0.01), and it is concluded that there is a statistically significant difference between the groups. Which statement is most consistent with this finding and conclusions?

A. The chance of making a type I error is 5 in 100.
B. The trial does not have enough power.
C. There is a high likelihood of having made a type II error.
D. The chance of making a type I error is 1 in 100.

4. You are reading a manuscript that evaluates the impact of obesity on enoxaparin pharmacokinetics. The authors used an unpaired t-test to compare the baseline values of body mass index (BMI) in normal subjects and obese subjects. You are evaluating the use of an unpaired t-test to compare the BMI between the two groups. Which choice best represents the most appropriate criteria to be met to use this parametric test?
A. The sample sizes in the normal and obese subjects should be equal to allow the use of a t-test.
B. A t-test is not appropriate because BMI data are ordinal.
C. The variance of the BMI data has to be similar in each group.
D. The pre-study power should be at least 90%.

5. You are evaluating the results and discussion of a journal club article to present to the pharmacy residents at your institution. The randomized, prospective, controlled trial evaluated the efficacy of a new controller drug for asthma. The primary end point was the morning forced expiratory volume in 1 second (FEV₁) in two groups of subjects (men and women). The difference in FEV₁ between the two groups was 15% (95% confidence interval [CI], 10%–21%). Which statement is most appropriate, given the results?
A. Without the reporting of a p-value, it is not possible to conclude whether these results were statistically significant.
B. There is a statistically significant difference between the men and women (p<0.05).
C. There is a statistically significant difference between the men and women (p<0.01).
D. There is no statistically significant difference between the men and women.

6. An early-phase clinical trial of 40 subjects evaluated a new drug known to increase high-density lipoprotein cholesterol (HDL) concentrations. The objective of the trial was to compare the new drug’s ability to increase HDL with that of lifestyle modifications (active control group). At the beginning of the study, the mean baseline HDL was 37 mg/dL in the active control group and 38 mg/dL in the new drug group. At the end of the 3-month trial, the mean HDL for the control group was 44 mg/dL and for the new drug group, 49 mg/dL. The p-value for the comparison at 3 months was 0.08. Which statement provides the best interpretation of these results?
A. An a priori α of less than 0.10 would have made the study more clinically useful.
B. The new drug and active control appear to be equally efficacious in increasing HDL concentrations.
C. The new drug is better than lifestyle modifications because it increases HDL concentrations to a greater extent.
D. This study is potentially underpowered.

7. Researchers planned a study to evaluate the percentage of subjects who achieved less than a target blood pressure (less than 140/90 mm Hg) when initiating therapy with two different doses of amlodipine. In the study of 100 subjects, the amlodipine 5-mg group (n=50) and the amlodipine 10-mg group (n=50) were compared. The investigators used a blood pressure goal as their primary end point, defined as the percentage of subjects who successfully achieved the blood pressure goal at 3 months. Which is the most appropriate statistical test to answer such a question?
A. Independent samples t-test.
B. Chi-square or Fisher exact test.
C. Wilcoxon signed-rank test.
D. One-sample t-test.

8. An investigational drug is being compared with an existing drug for the treatment of anemia in patients with chronic kidney disease. The study is designed to detect a minimum 20% difference in response rates between the groups, if one exists, with an a priori α of 0.05 or less. The investigators are unclear whether the 20% difference between response rates is too large and think a smaller difference might be more clinically meaningful. In revising their study, they decide they want to be able to detect a minimum 10% difference in response. Which change to the study parameters is most appropriate?
A. Increase the sample size.
B. Select an α of 0.001 as a cutoff for statistical significance.
C. Select an α of 0.10 as a cutoff for statistical significance.
D. Decrease the sample size.
9. You are designing a new computer alert system to investigate the impact of several factors on the risk of corrected QT interval (QTc) prolongation. You want to develop a model to predict which patients are most likely to experience QTc prolongation after the administration of certain drugs or the presence of certain conditions. You plan to assess the presence or absence of several different variables. Which technique will be most useful in completing such an analysis?

A. Correlation.
B. Kaplan-Meier curve.
C. Regression.
D. Confidence intervals.
I. INTRODUCTION TO STATISTICS

A. Method for Collecting, Classifying, Summarizing, and Analyzing Data

B. Useful Tool for Quantifying Clinical and Laboratory Data in a Meaningful Way

C. Assists in Determining Whether and by How Much a Treatment or Procedure Affects a Group of Patients

D. Why Pharmacists Need to Know Statistics

E. As Statistics Pertains to Most of You
   1. Pharmacotherapy Specialty Examination content outline, Domain 2: Drug Information and Evidence Based Medicine (25%)
   2. Task statements:
      a. Retrieve information that addresses pharmacotherapy-related inquiries in order to optimize patient care.
      b. Evaluate pharmacotherapy-related literature, databases, and health information in order to translate findings into practice.
      c. Conduct pharmacotherapy-related research using appropriate scientific principles in order to ensure optimal patient care.
      d. Disseminate pharmacotherapy-related information or research in order to educate health care professionals and trainees.

F. Examples of Online Statistical and Study Design Tools
   1. www.graphpad.com/quickcalcs/

G. Several Papers Have Investigated the Various Types of Statistical Tests Used in the Biomedical Literature; the data from one of these papers are illustrated in the text that follows. Tables 1 and 2 are modified from Windish DM, Huot SJ, Green ML. Medicine resident’s understanding of the biostatistics and results in the medical literature. JAMA 2007;298:1010-22.
Table 1. Statistical Content of Original Articles in *New England Journal of Medicine*, 2004–2005

<table>
<thead>
<tr>
<th>Statistical Procedure</th>
<th>% of Articles Containing Methods</th>
<th>Statistical Procedure</th>
<th>% of Articles Containing Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>No statistics or descriptive statistics</td>
<td>13</td>
<td>Adjustment and standardization</td>
<td>1</td>
</tr>
<tr>
<td>t-tests</td>
<td>26</td>
<td>Multiway tables</td>
<td>13</td>
</tr>
<tr>
<td>Contingency tables</td>
<td>53</td>
<td>Power analyses</td>
<td>39</td>
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<tr>
<td>Nonparametric tests</td>
<td>27</td>
<td>Cost-benefit analysis</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Epidemiologic statistics</td>
<td>35</td>
<td>Sensitivity analysis</td>
<td>6</td>
</tr>
<tr>
<td>Pearson correlation</td>
<td>3</td>
<td>Repeated-measures analysis</td>
<td>12</td>
</tr>
<tr>
<td>Simple linear regression</td>
<td>6</td>
<td>Missing-data methods</td>
<td>8</td>
</tr>
<tr>
<td>Analysis of variance</td>
<td>16</td>
<td>Noninferiority trials</td>
<td>4</td>
</tr>
<tr>
<td>Transformation</td>
<td>10</td>
<td>Receiver operating characteristics</td>
<td>2</td>
</tr>
<tr>
<td>Nonparametric correlation</td>
<td>5</td>
<td>Resampling</td>
<td>2</td>
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<tr>
<td>Survival methods</td>
<td>61</td>
<td>Principal component and cluster analyses</td>
<td>2</td>
</tr>
<tr>
<td>Multiple regression</td>
<td>51</td>
<td>Other methods</td>
<td>4</td>
</tr>
<tr>
<td>Multiple comparisons</td>
<td>23</td>
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</tr>
</tbody>
</table>

Table 2. Statistical Content of Original Articles from Six Major Medical Journals from January to March 2005 (n=239 articles)*

<table>
<thead>
<tr>
<th>Statistical Test</th>
<th>No. (%)</th>
<th>Statistical Test</th>
<th>No. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptive statistics (mean, median, frequency, SD, and IQR)</td>
<td>219 (91.6)</td>
<td>Others</td>
<td></td>
</tr>
<tr>
<td>Simple statistics</td>
<td>120 (50.2)</td>
<td>Intention-to-treat analysis</td>
<td>42 (17.6)</td>
</tr>
<tr>
<td>Chi-square analysis</td>
<td>70 (29.3)</td>
<td>Incidence or prevalence</td>
<td>39 (16.3)</td>
</tr>
<tr>
<td>t-test</td>
<td>48 (20.1)</td>
<td>Relative risk or risk ratio</td>
<td>29 (12.2)</td>
</tr>
<tr>
<td>Kaplan-Meier analysis</td>
<td>48 (20.1)</td>
<td>Sensitivity analysis</td>
<td>21 (8.8)</td>
</tr>
<tr>
<td>Wilcoxon rank sum test</td>
<td>38 (15.9)</td>
<td>Sensitivity or specificity</td>
<td>15 (6.3)</td>
</tr>
<tr>
<td>Fisher exact test</td>
<td>33 (13.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis of variance</td>
<td>21 (8.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>16 (6.7)</td>
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<tr>
<td>Multivariate analysis</td>
<td>164 (68.6)</td>
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<tr>
<td>Cox proportional hazards</td>
<td>64 (26.8)</td>
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<tr>
<td>Multiple logistic regression</td>
<td>54 (22.6)</td>
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<tr>
<td>Multiple linear regression</td>
<td>7 (2.9)</td>
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<tr>
<td>Other regression analysis</td>
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</tr>
<tr>
<td>None</td>
<td>5 (2.1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


IQR = interquartile range; SD = standard deviation.
II. TYPES OF VARIABLES AND DATA

A. Definition: Random Variables—A variable with observed values that may be considered outcomes of an experiment and whose values cannot be anticipated with certainty before the experiment is conducted

B. Two Types of Random Variables
   1. Discrete variables (e.g., dichotomous, categorical)
   2. Continuous variables

C. Discrete Variables
   1. Can take only a limited number of values within a given range
   2. Nominal: Classified into groups in an unordered manner and with no indication of relative severity (e.g., male or female sex, mortality [dead or alive], disease presence [yes or no], race, marital status)
   3. Ordinal: Ranked in a specific order but with no consistent level of magnitude of difference between ranks (e.g., NYHA functional class describes the functional status of patients with heart failure, and subjects are classified in increasing order of symptoms: I, II, III, IV; Likert-type scales)
   4. Common error: Measure of central tendency—In most cases, means and standard deviations (SDs) should not be reported with ordinal data. What is a common incorrect use of means and SDs to show ordinal data?

D. Continuous Variables, Sometimes Called Counting Variables
   1. Continuous variables can take on any value within a given range.
   2. Interval: Data are ranked in a specific order with a consistent change in magnitude between units; the zero point is arbitrary (e.g., degrees Fahrenheit).
   3. Ratio: Like “interval” but with an absolute zero (e.g., degrees Kelvin, heart rate, blood pressure, time, distance)

III. TYPES OF STATISTICS

A. Descriptive Statistics: Used to summarize and describe data that are collected or generated in research studies. This is done both visually and numerically.
   1. Visual methods of describing data
      a. Frequency distribution
      b. Histogram
      c. Scatterplot
      d. Boxplot
   2. Numerical methods of describing data: Measures of central tendency
      a. Arithmetic mean (i.e., average)
         i. Sum of all values divided by the total number of values
         ii. Should generally be used only for continuous and normally distributed data
         iii. Very sensitive to outliers and tend toward the tail, which has the outliers
         iv. Most commonly used and most understood measure of central tendency
         v. Geometric mean
      b. Median
         i. Midpoint of the values when placed in order from highest to lowest. Half of the observations are above and half are below. When there is an even number of observations, it is the mean of the two middle values.
         ii. Also called 50th percentile
         iii. Can be used for ordinal or continuous data (especially good for skewed populations)
         iv. Insensitive to outliers

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c. Mode
  i. Most common value in a distribution
  ii. Can be used for nominal, ordinal, or continuous data
  iii. Sometimes, there may be more than one mode (e.g., bimodal, trimodal).
  iv. Does not help describe meaningful distributions with a large range of values, each of which
  occurs infrequently

3. Numerical methods of describing data: Measures of data spread or variability
   a. Standard deviation
     i. Measure of the variability about the mean; most common measure used to describe the spread
        of data
     ii. Square root of the variance (average squared difference of each observation from the mean);
        returns variance back to original units (non-squared)
     iii. Appropriately applied only to continuous data that are normally or near normally distributed
        or that can be transformed to be normally distributed
     iv. By the empirical rule for normal distributions, 68% of the sample values are found within
        ±1 SD, 95% are found within ±2 SD, and 99% are found within ±3 SD.
     v. The coefficient of variation relates the mean and the SD (SD/mean × 100%).
   b. Range
     i. Difference between the smallest and largest values in a data set does not give a tremendous
        amount of information by itself.
     ii. Easy to compute (simple subtraction)
     iii. Size of range is very sensitive to outliers.
     iv. Often reported as the actual values rather than the difference between the two extreme values
   c. Percentiles
     i. The point (value) in a distribution in which a value is larger than some percentage of the other
        values in the sample. Can be calculated by ranking all data in a data set
     ii. The 75th percentile lies at a point at which 75% of the other values are smaller.
     iii. Does not assume the population has a normal distribution (or any other distribution)
     iv. The interquartile range (IQR) is an example of the use of percentiles to describe the middle
        50% values. The IQR encompasses the 25th–75th percentile.

4. Presenting data using only measures of central tendency can be misleading without some idea of data
   spread. Studies that report only medians or means without their accompanying measures of data spread
   should be closely scrutinized. What are the measures of spread that should be used with means and
   medians?

5. Example data set (Table 3)

Table 3. Twenty Baseline HDL Concentrations from an Experiment Evaluating the Impact of Green Tea on HDL

<table>
<thead>
<tr>
<th></th>
<th>64</th>
<th>60</th>
<th>59</th>
<th>65</th>
<th>64</th>
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</thead>
<tbody>
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<td>59</td>
<td>65</td>
<td>87</td>
<td>49</td>
<td>46</td>
<td>46</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. Calculate the mean, median, and mode of the data set given in Table 3.
   b. Calculate the range, and SD (on examination, you will not have to do this by hand).
   c. Evaluate the visual presentation of the data.
B. Inferential Statistics
1. Conclusions or generalizations made about a population (large group) from the study of a sample of that population.
2. Choosing and evaluating statistical methods depend, in part, on the type of data used.
3. An educated statement about an unknown population is commonly called an inference in statistics.
4. Statistical inference can be made by estimation or hypothesis testing.

IV. POPULATION DISTRIBUTIONS

A. Discrete Distributions
1. Binomial distribution
2. Poisson distribution

B. Normal (Gaussian) Distribution
1. Most common model for population distributions
2. Symmetric or bell-shaped frequency distribution
3. Landmarks for continuous, normally distributed data
   a. $\mu$: Population mean is equal to zero.
   b. $\sigma$: Population SD is equal to 1.
   c. $x$ and $s$: These represent the sample mean and SD.
4. When a random variable is measured in a large enough sample of any population, some values will occur more often than will others.
5. A visual check of a distribution can help determine whether it is normally distributed (whether it appears symmetric and bell shaped). Need the data to perform these checks.
   a. Frequency distribution and histograms (visually look at the data; you should do this anyway)
   b. Median and mean will be about equal for normally distributed data (most practical and easiest to use).
   c. Formal test: Kolmogorov-Smirnov test
   d. More challenging to evaluate this when we do not have access to the data (when we are reading an article), because most articles do not present all data or both the mean and median
6. The parameters mean and SD define a normally distributed population.
7. Probability: The likelihood that any one event will occur given all the possible outcomes
8. Estimation and sampling variability
   a. One method that can be used to make an inference about a population parameter
   b. Separate samples (even of the same size) from a single population will give slightly different estimates.
   c. The distribution of means from random samples approximates a normal distribution.
      i. The mean of this “distribution of means” is equal to the unknown population mean, $\mu$.
      ii. The SD of the means is estimated by the standard error of the mean (SEM).
      iii. As in any normal distribution, 95% of the sample means lie within $\pm 2$ SEM of the population mean.
   d. The distribution of means from these random samples is about normal regardless of the underlying population distribution (central limit theorem). You will get slightly different mean and SD values each time you repeat this experiment.
   e. The SEM is estimated with a single sample by dividing the SD by the square root of the sample size (n). The SEM quantifies uncertainty in the estimate of the mean, not variability in the sample. Important for hypothesis testing and 95% CI estimation
f. Why is all this information about the difference between the SEM and SD worth knowing?
   i. Calculation of CIs. (95% CI is approximately the mean ± 2 times the SEM.)
   ii. Hypothesis testing
   iii. Deception (e.g., makes results look less “variable,” especially when used in graphic format)

9. Recall the previous example about HDL and green tea. From the calculated values in section III, do these data appear to be normally distributed?

V. CONFIDENCE INTERVALS

A. Commonly Reported as a Way to Estimate a Population Parameter
   1. In the medical literature, 95% CIs are the most commonly reported CIs. In repeated samples, 95% of all CIs include true population value (i.e., the likelihood or confidence [or probability] that the population value is contained within the interval). In some cases, 90% or 99% CIs are reported. Why are 95% CIs most often reported?
   2. Example
      a. Assume a baseline birth weight in a group (n=51) with a mean ± SD of 1.18 ± 0.4 kg.
      b. 95% CI is about equal to the mean ± 1.96 × SEM (or 2 × SEM). In reality, it depends on the distribution being used and is a bit more complicated.
      c. What is the 95% CI? The 95% CI is calculated to be (1.07, 1.29), meaning there is 95% certainty that the true mean of the entire population studied will be 1.07–1.29 kg.
      d. What is the 90% CI? The 90% CI is calculated to be (1.09, 1.27). The 95% CI will always be wider than the 90% CI for any given sample. Therefore, the wider the CI, the more likely it is to encompass the true population mean.
   3. The differences between the SD, SEM, and CIs should be noted when interpreting the literature because they are often used interchangeably. Although it is common for CIs to be confused with SDs, the information each provides is quite different and has to be assessed correctly.
   4. Recall the previous example about HDL and green tea. What is the 95% CI of the data set, and what does that mean?

B. CIs Can Also Be Used for Any Sample Estimate. Estimates derived from categorical data such as risk, risk differences, and risk ratios are often presented with the CI and will be discussed in the text that follows.

C. CIs Instead of Hypothesis Testing
   1. Hypothesis testing and calculation of p-values tell us (ideally) whether there is or is not a statistically significant difference between groups, but they do not tell us anything about the magnitude of the difference.
   2. CIs help us determine the importance of a finding or findings, which we can apply to a situation.
   3. CIs give us an idea of the magnitude of the difference between groups and the statistical significance.
   4. CIs are a range of data, together with a point estimate of the difference.
   5. Wide CIs
      a. Many results are possible, either larger or smaller than the point estimate provided by the study.
      b. All values contained in the CI are statistically plausible.
   6. If the estimate is the difference between two continuous variables: A CI that includes zero (no difference between two variables) can be interpreted as not statistically significant (a p-value of 0.05 or greater). There is no need to show both the 95% CI and the p-value.
7. The interpretation of CIs for odds ratios and relative risks is somewhat different. In that case, a value of 1 indicates no difference in risk, and if the CI includes 1, there is no statistical difference. (See the discussion of case-control/cohort in other sections for how to interpret CIs for odds ratios and relative risks.)

VI. HYPOTHESIS TESTING

A. Null and Alternative Hypotheses (see Table 4 for other types of examples)

1. Null hypothesis (H₀): Example: No difference between groups being compared (treatment A equals treatment B)
2. Alternative hypothesis (Hₐ): Example: Opposite of null hypothesis; states that there is a difference (treatment A does not equal treatment B)
3. The structure or the manner in which the hypothesis is written dictates which statistical test is used. Two-sample t-test: H₀: Mean 1 = Mean 2
4. Used to assist in determining whether any observed differences between groups can be explained by chance
5. Tests for statistical significance (hypothesis testing) determine whether the data are consistent with H₀ (no difference).
6. The results of the hypothesis testing will indicate whether enough evidence exists for H₀ to be rejected.
   a. If H₀ is rejected: Statistically significant difference between groups (unlikely attributable to chance)
   b. If H₀ is not rejected: No statistically significant difference between groups (any apparent differences may be attributable to chance). Note that we are not concluding that the treatments are equal.
7. Types of hypothesis testing. These are situations in which two groups are being compared. There are numerous other examples of situations these procedures could be applied to (Table 4).

Table 4. Types of Hypothesis Testing

<table>
<thead>
<tr>
<th>Question</th>
<th>Hypothesis</th>
<th>Method</th>
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<tbody>
<tr>
<td>Nondirectional</td>
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<tr>
<td>Difference</td>
<td>Are the means different?</td>
<td>H₀: Mean₁ = Mean₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hₐ: Mean₁ ≠ Mean₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H₀: Mean₁ – Mean₂ = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hₐ: Mean₁ – Mean₂ ≠ 0</td>
</tr>
<tr>
<td>Equivalence</td>
<td>Are the means practically equivalent?</td>
<td>H₀: Mean₁ – Mean₂ ≥ Δ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hₐ: Mean₁ – Mean₂ &lt; Δ</td>
</tr>
<tr>
<td>Directional</td>
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<td></td>
</tr>
<tr>
<td>Superiority</td>
<td>Is mean 1 &gt; mean 2? (or some other similarly worded question)</td>
<td>H₀: Mean₁ ≤ Mean₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hₐ: Mean₁ &gt; Mean₂</td>
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<td>or</td>
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<td>H₀: Mean₁ – Mean₂ ≤ 0</td>
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<tr>
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<td></td>
<td>Hₐ: Mean₁ – Mean₂ &gt; 0</td>
</tr>
<tr>
<td>Noninferiority</td>
<td>Is mean 1 no more than a certain amount lower than mean 2?</td>
<td>H₀: Mean₁ – Mean₂ ≥ Δ</td>
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<tr>
<td></td>
<td></td>
<td>Hₐ: Mean₁ – Mean₂ &lt; Δ</td>
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</tbody>
</table>

Δ = equivalence or noninferiority margin; H₀ = null hypothesis; Hₐ = alternative hypothesis.
B. To Determine What Is Sufficient Evidence to Reject $H_0$: Set the a priori significance level ($\alpha$) and generate the decision rule.
   1. Developed after the research question has been stated in hypothesis form
   2. Used to determine the level of acceptable error caused by a false positive (also known as level of significance)
      a. Convention: A priori $\alpha$ is usually 0.05.
      b. Critical value is calculated, capturing how extreme the sample data must be to reject $H_0$.

C. Perform the Experiment and Estimate the Test Statistic.
   1. A test statistic is calculated from the observed data in the study, which is compared with the critical value.
   2. Depending on this test statistic’s value, $H_0$ is not rejected (often called fail to reject) or rejected.
   3. In general, the test statistic and critical value are not presented in the literature; instead, $p$-values are generally reported and compared with a priori $\alpha$ values to assess statistical significance. $p$-value: Probability of obtaining a test statistic and critical value as extreme as or more extreme than the one actually obtained.
   4. Because computers are used in these tests, this step is often transparent; the $p$-value estimated in the statistical test is compared with the a priori $\alpha$ (usually 0.05), and the decision is made.

VII. STATISTICAL TESTS AND CHOOSING A STATISTICAL TEST

A. Which Tests Do You Need to Know?

B. Choosing the Appropriate Statistical Test Depends on the Following:
   1. Type of data (nominal, ordinal, or continuous)
   2. Distribution of data (e.g., normal)
   3. Number of groups
   4. Study design (e.g., parallel, crossover)
   5. Presence of confounding variables
   6. One-tailed versus two-tailed
   7. Parametric versus nonparametric tests
      a. Parametric tests assume the following:
         i. Data being investigated have an underlying distribution that is normal or close to normal or, more correctly, randomly drawn from a parent population with a normal distribution. Remember how to estimate this (mean ~ median)?
         ii. Data measured are continuous data, measured on either an interval or a ratio scale.
         iii. Parametric tests assume that the data being investigated have variances that are homogeneous between the groups investigated. This is often called homoscedasticity.
      b. Nonparametric tests are used when data are not normally distributed or do not meet other criteria for parametric tests (e.g., discrete data).
C. Parametric Tests

1. Student t-test: Several different types
   a. One-sample test: Compares the mean of the study sample with the population mean

   | Group 1 | Known population mean |
   |

   b. Two-sample, independent samples, or unpaired test: Compares the means of two independent samples. This is an independent samples test.

   | Group 1 | Group 2 |
   |

   i. Equal variance test
      (a) Rule for variances: If the ratio of larger variance to smaller variance is greater than 2, we generally conclude the variances are different.
      (b) Formal test for differences in variances: F test
      (c) Adjustments can be made for cases of unequal variance.

   ii. Unequal variance

   c. Paired test: Compares the mean difference of paired or matched samples. This is a related samples test.

<table>
<thead>
<tr>
<th>Group 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement 1</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

d. Common error: Use of multiple t-tests with more than two groups

2. Analysis of variance (ANOVA): A more generalized version of the t-test that can apply to more than two groups
   a. One-way ANOVA: Compares the means of three or more groups in a study; also known as single-factor ANOVA. This is an independent samples test.

   | Group 1 | Group 2 | Group 3 |
   |

   b. Two-way ANOVA: Additional factor (e.g., age) added

   | Young groups | Group 1 | Group 2 | Group 3 |
   | Old groups   | Group 1 | Group 2 | Group 3 |

   c. Repeated-measures ANOVA: This is a related samples test.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Related Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement 1</td>
<td>Measurement 2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Several more complex factorial ANOVAs can be used.

e. Many comparison procedures are used to determine which groups actually differ from each other.

   Post hoc tests: Tukey HSD (Honestly Significant Difference), Bonferroni, Scheffé, Newman-Keuls

3. Analysis of covariance (ANCOVA): Provides a method to explain the influence of a categorical variable (independent variable) on a continuous variable (dependent variable) while statistically controlling for other variables (confounding)
D. Nonparametric Tests
1. These tests may also be used for continuous data that do not meet the assumptions of the t-test or ANOVA.
2. Tests for independent samples
   a. Wilcoxon rank sum test, Mann-Whitney \( U \) test, or Wilcoxon Mann-Whitney test: Compare two independent samples (related to a t-test)
   b. Kruskal-Wallis one-way ANOVA by ranks
      i. Compares three or more independent groups (related to one-way ANOVA)
      ii. Post hoc testing
3. Tests for related or paired samples
   a. Sign test and Wilcoxon signed-rank test: Compares two matched or paired samples (related to a paired t-test)
   b. Friedman ANOVA by ranks: Compares three or more matched or paired groups

E. Nominal Data
1. Chi-square (\( \chi^2 \)) test: Compares expected and observed proportions between two or more groups
   a. Test of independence
   b. Test of goodness of fit
2. Fisher exact test: Specialized version of the chi-square test for small groups (cells) containing less than five predicted observations
3. McNemar: Paired samples
4. Mantel-Haenszel: Controls for the influence of confounders

F. Correlation and Regression (see section IX)

G. Choosing the Most Appropriate Statistical Test: Example 1
1. A trial was conducted to determine the efficacy and safety of alirocumab in reducing lipids and cardiovascular events. Alirocumab plus statins was compared with placebo plus statins regarding their effect on low-density lipoprotein cholesterol (LDL) concentrations. The trial was designed such that the subjects’ baseline characteristics were as comparable as possible with each other. The intended primary end point for this trial was the difference in LDL between the two treatments at week 24. The full trial is published: N Engl J Med 2015;372:1489-99. Note that only partial results are presented. The results of the trial are reported as follows:

| Table 5. Baseline Characteristics and Alirocumab and Placebo Effect on LDL* |
|-----------------------------|-----------------------------|-----------------------------|
|                             | Alirocumab plus Statins     | Placebo plus Statins         |
|                             | \((n=1553)\)                | \((n=788)\)                  |
| Men/women                   | (63.3%) 983/570             | (60.2%) 474/314              |
| Smokers                     | (20.9%) 325/1228            | (20.2%) 159/629              |
| Baseline LDL, mg/dL         | 122.8 ± 42.7                | 122.0 ± 41.6                 |
| Final LDL, mg/dL            | 48.3 ± 35.2                 | 118.9 ± 33.5                 |

*Data are presented as mean ± SD.
2. Which is the appropriate statistical test to determine baseline differences in the following:
   a. Sex distribution?
   b. LDL?
   c. Percentage of smokers and nonsmokers?
3. Which is the appropriate statistical test to determine the following:
   a. The effect of alirocumab plus statins on LDL?
   b. The primary end point?

VIII. DECISION ERRORS

Table 6. Summary of Decision Errors

<table>
<thead>
<tr>
<th>Test Result</th>
<th>Underlying Truth or Reality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept $H_0$ (no difference)</td>
<td>$H_0$ is true (no difference)</td>
</tr>
<tr>
<td>Reject $H_0$ (difference)</td>
<td>Type I error ($\alpha$ error)</td>
</tr>
</tbody>
</table>

$H_0$ = null hypothesis.

A. Type I Error: The probability of making this error is defined as the significance level $\alpha$.
   1. Convention is to set the $\alpha$ to 0.05, effectively meaning that, 1 in 20 times, a type I error will occur when the $H_0$ is rejected. Thus, 5.0% of the time, a researcher will conclude that there is a statistically significant difference when one does not actually exist.
   2. The calculated chance that a type I error has occurred is called the p-value.
   3. The p-value tells us the likelihood of obtaining a given (or a more extreme) test result if the $H_0$ is true. When the $\alpha$ level is set a priori, $H_0$ is rejected when p is less than $\alpha$. In other words, the p-value tells us the probability of being wrong when we conclude that a true difference exists (false positive).
   4. A lower p-value does not mean the result is more important or more meaningful but only that it is statistically significant and not likely to be attributable to chance.

B. Type II Error: The probability of making this error is called beta.
   1. Concluding that no difference exists when one truly does (not rejecting $H_0$ when it should be rejected)
   2. It has become a convention to set $\beta$ at 0.20–0.10.

C. Power ($1 - \beta$)
   1. The probability of making a correct decision when $H_0$ is false; the ability to detect differences between groups if one actually exists
   2. Dependent on the following factors:
      a. Predetermined $\alpha$
      b. Sample size
      c. The size of the difference between the outcomes you want to detect. Often not known before conducting the experiment, so to estimate the power of your test, you will have to specify how large a change is worth detecting
      d. The variability of the outcomes that are being measured
      e. Items c and d are generally determined from previous data or the literature.
3. Power is decreased by the following (in addition to the earlier criteria):
   a. Poor study design
   b. Incorrect statistical tests (use of nonparametric tests when parametric tests are appropriate)

4. Statistical power analysis and sample size calculation
   a. Related to the previous discussion of power and sample size
   b. Sample size estimates should be performed in all studies a priori.
   c. Necessary components for estimating appropriate sample size
      i. Acceptable type II error rate (usually 0.10–0.20)
      ii. Observed difference in predicted study outcomes that is clinically significant
      iii. The expected variability in item ii
      iv. Acceptable type I error rate (usually 0.05)
      v. Statistical test that will be used for primary end point

5. Statistical significance versus clinical significance
   a. As stated earlier, the size of the p-value is not necessarily related to the clinical importance of the result. Smaller values mean only that chance is less likely to explain observed differences.
   b. Statistically significant does not necessarily mean clinically significant.
   c. Lack of statistical significance does not mean that results are not clinically important.
   d. When considering nonsignificant findings, consider sample size, estimated power, and observed variability.

IX. CORRELATION AND REGRESSION

A. Introduction: Correlation vs. Regression
   1. Correlation examines the strength of the association between two variables. It does not necessarily assume that one variable is useful in predicting the other.
   2. Regression examines the ability of one or more variables to predict another variable.

B. Pearson Correlation
   1. The strength of the relationship between two variables that are normally distributed, ratio or interval scaled, and linearly related is measured with a correlation coefficient.
   2. Often called the degree of association between the two variables
   3. Does not necessarily imply that one variable is dependent on the other (regression analysis will do that)
   4. Pearson correlation (r) ranges from −1 to +1 and can take any value in between:

<table>
<thead>
<tr>
<th>−1</th>
<th>0</th>
<th>+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect negative linear relationship</td>
<td>No linear relationship</td>
<td>Perfect positive linear relationship</td>
</tr>
</tbody>
</table>

   5. Hypothesis testing is performed to determine whether the correlation coefficient is different from zero. This test is highly influenced by sample size.

C. Pearls About Correlation
   1. The closer the magnitude of r to 1 (either + or −), the more highly correlated the two variables. The weaker the relationship between the two variables, the closer r is to 0.
   2. There is no agreed-on or consistent interpretation of the value of the correlation coefficient. It is dependent on the environment of the investigation (laboratory vs. clinical experiment).
3. Pay more attention to the magnitude of the correlation than to the p-value because it is influenced by sample size.

4. Crucial to the proper use of correlation analysis is the interpretation of the graphic representation of the two variables. Before using correlation analysis, it is essential to generate a scatterplot of the two variables to visually examine the relationship.

D. Spearman Rank Correlation: Nonparametric test that quantifies the strength of an association between two variables but does not assume a normal distribution of continuous data. Can be used for ordinal data or non-normally distributed continuous data

E. Regression

1. A statistical technique related to correlation. There are many different types. For simple linear regression, one continuous outcome (dependent) variable and one continuous independent (causative) variable

2. Two main purposes of regression: Development of prediction model and accuracy of prediction

3. Prediction model: Making predictions of the dependent variable from the independent variable; \( Y = mx + b \) (dependent variable = slope \( \times \) independent variable + intercept)

4. Accuracy of prediction: How well the independent variable predicts the dependent variable. Regression analysis determines the extent of variability in the dependent variable that can be explained by the independent variable.
   a. Coefficient of determination \( (r^2) \) measured describing this relationship. Values of \( r^2 \) can range from 0 to 1.
   b. An \( r^2 \) of 0.80 could be interpreted as saying that 80% of the variability in \( Y \) is explained by the variability in \( X \).
   c. This does not provide a mechanistic understanding of the relationship between \( X \) and \( Y \) but rather a description of how clearly such a model (linear or otherwise) describes the relationship between the two variables.
   d. Like the interpretation of \( r \), the interpretation of \( r^2 \) is dependent on the scientific arena (e.g., clinical research, basic research, social science research) to which it is applied.

5. For simple linear regression, two statistical tests can be used.
   a. To test the hypothesis that the \( y \)-intercept differs from zero
   b. To test the hypothesis that the slope of the line is different from zero

6. Regression is useful in constructing predictive models. The literature is full of examples of predictions. The process involves developing a formula for a regression line that best fits the observed data.

7. Like correlation, there are many different types of regression analysis.
   a. Multiple linear regression: One continuous independent variable and two or more continuous dependent variables
   b. Simple logistic regression: One categorical response (dependent) variable and one continuous or categorical explanatory (independent) variable
   c. Multiple logistic regression: One categorical response (dependent) variable and two or more continuous or categorical explanatory (independent) variables
   d. Nonlinear regression: Variables are not linearly related (or cannot be transformed into a linear relationship). This is where our pharmacokinetic equations come from.
   e. Polynomial regression: Any number of response and continuous variables with a curvilinear relationship (e.g., cubed, squared)
8. Example of regression
   a. The following data are taken from a study evaluating enoxaparin use. The authors were interested in predicting patient response (measured as anti-factor Xa concentrations) from the enoxaparin dose in the 75 subjects who were studied.

<table>
<thead>
<tr>
<th>Enoxaparin Dose (mg/Kg)</th>
<th>Antifactor Xa Concentrations (U/mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20</td>
<td>0.00</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.80</td>
<td>2.00</td>
</tr>
<tr>
<td>0.60</td>
<td>3.00</td>
</tr>
<tr>
<td>0.40</td>
<td>4.00</td>
</tr>
<tr>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

   Figure 1. Relationship between antifactor Xa concentrations and enoxaparin dose.

   b. The authors performed regression analysis and reported the following: Slope: 0.227, y-intercept: 0.097, p<0.05, r² = 0.31.

   c. Answer the following questions:
      i. What are the assumptions necessary to use regression analysis?
      ii. Provide an interpretation of the coefficient of determination.
      iii. Predict anti-factor Xa concentrations at enoxaparin doses of 2 and 3.75 mg/kg.
      iv. What does the p<0.05 value indicate?

X. SURVIVAL ANALYSIS

   A. Studies the Time Between Entry in a Study and Some Event (e.g., death, myocardial infarction)
      1. Censoring makes survival methods unique; considers that some subjects leave the study for reasons other than the event (e.g., lost to follow-up, end of study period)
      2. Considers that all subjects do not enter the study at the same time
      3. Standard methods of statistical analysis such as t-tests and linear or logistic regression may not be appropriately applied to survival data because of censoring.
B. Estimating the Survival Function

1. Kaplan-Meier method
   a. Uses survival times (or censored survival times) to estimate the proportion of people who would survive a given length of time under the same circumstances
   b. Allows the production of a table (life table) and a graph (survival curve)
   c. We can visually evaluate the curves, but we need a test to evaluate them formally.

2. Log-rank test: Compare the survival distributions between two or more groups.
   a. This test precludes an analysis of the effects of several variables or the magnitude of difference between groups or the CI (see the text that follows for the Cox proportional hazards model).
   b. \( H_0: \) No difference in survival between the two populations
   c. Log-rank test uses several assumptions.
      i. Random sampling and subjects chosen independently
      ii. Consistent criteria for entry or end point
      iii. Baseline survival rate does not change as time progresses.
      iv. Censored subjects have the same average survival time as uncensored subjects.

3. Cox proportional hazards model
   a. Most popular method to evaluate the impact of covariates; reported (graphically) like Kaplan-Meier
   b. Investigates several variables at a time
   c. Actual method of construction and calculation is complex.
   d. Compares survival in two or more groups after adjusting for other variables
   e. Allows calculation of a hazard ratio (and CI)

XI. SELECTED REPRESENTATIVE STATISTICAL TESTS

Table 7. Representative Statistical Tests

<table>
<thead>
<tr>
<th>Type of Variable</th>
<th>2 Groups (independent)</th>
<th>2 Groups (related)</th>
<th>&gt; 2 Groups (independent)</th>
<th>&gt; 2 Groups (related)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>( \chi^2 ) or Fisher exact test</td>
<td>McNemar test</td>
<td>( \chi^2 )</td>
<td>Cochran ( Q )</td>
</tr>
<tr>
<td>Ordinal</td>
<td>Wilcoxon rank sum Mann-Whitney ( U ) test</td>
<td>Wilcoxon signed-rank Sign test</td>
<td>Kruskal-Wallis (MCP)</td>
<td>Friedman ANOVA</td>
</tr>
<tr>
<td>Continuous</td>
<td>Equal variance t-test Unequal variance t-test</td>
<td>Paired t-test</td>
<td>One-way ANOVA (MCP)</td>
<td>Repeated-measures ANOVA</td>
</tr>
<tr>
<td>1 factor</td>
<td>ANCOVA</td>
<td>Two-way repeated-measures ANOVA</td>
<td>Two-way ANOVA (MCP)</td>
<td>Two-way repeated-measures ANOVA</td>
</tr>
</tbody>
</table>

ANCOVA = analysis of covariance; ANOVA = analysis of variance; MCP = multiple-comparisons procedure.
REFERENCES

1. **Answer: A**
The NYHA functional class is an ordinal scale from I (no symptoms) to IV (severe symptoms). Neither ANOVA nor ANCOVA is appropriate for ordinal or noncontinuous data (Answers C and D are incorrect). The Wilcoxon signed-rank test is an appropriate nonparametric test to use for paired ordinal data, such as the change in NYHA functional class over time on the same person (Answer B is incorrect). The Kruskal-Wallis test is the nonparametric analog of a one-way ANOVA and is appropriate for this analysis (Answer A is correct).

2. **Answer: C**
You cannot determine which finding is more important (in this case, the best drug) on the basis of the p-value (i.e., a lower p-value does not mean more important) (Answer B is incorrect). All statistically significant results are interpreted as significant without respect to the size of the p-value. This trial had four independent samples, and use of the unpaired (independent samples) t-test is not appropriate because it requires several unnecessary tests and increases the chances of making a type I error (Answer A is incorrect). In this setting, ANOVA is the correct test (Answer C is correct), followed by a multiple-comparisons procedure to determine where the actual differences between groups lie. A paired t-test is inappropriate because this is a parallel-group trial (Answer D is incorrect). Use of ANOVA in this case assumes a normal distribution and equal variance in each of the four groups.

3. **Answer: D**
The typical a priori α error (type I) rate is 5% (i.e., when the study was designed, the error rate was designed to be 5% or less). The actual type I error rate is reported in the question as 0.01 (1%) (Answer A is incorrect). Answers B and C are related; the study did have enough power because a statistically significant difference was observed. Similarly, a type II error was not made because this error has to do with not finding a difference when one truly exists. In this question, the type I error rate is 1%, the value of the p-value (Answer D is correct).

4. **Answer: C**
Sample sizes need not be equal to use a t-test (Answer A is incorrect). Body mass index data are not ordinal but continuous; thus, a t-test is appropriate (Answer B is incorrect). The assumption of equal variances is required to use any parametric test (Answer C is correct). A specific value for power is not required to use a test (Answer D is incorrect).

5. **Answer: B**
The reporting of the mean difference and CI is thought by many to be a superior means of presenting the results from a clinical trial because it describes both precision and statistical significance, as compared with a p-value, which distills everything into one value, making Answer A incorrect. The presentation of the data in this manner clearly shows all the necessary information for making the appropriate conclusion. To assess statistical significance by use of CIs, the 95% CI (corresponding to the 5% type I error rate used in most studies) may not contain zero (signifying no difference between men and women) for the mean difference, making Answer D incorrect. Answer B is correct because the p-value of less than 0.05 corresponds to the 95% CI in that item. To evaluate Answer C, we would need to know the 99% CI.

6. **Answer: D**
Answer A is incorrect because it uses unconventional approaches to determine statistical significance. Although this can be done, it is unlikely to be accepted by other readers and investigators. This study observed a nonsignificant increase in HDL concentration between the two groups. With a small sample size, such as the one used in this study, there is always concern about adequate power to observe a difference between the two treatments. A difference may exist between these two drugs, but the number of subjects studied may be too small to detect it statistically. Answer D is correct because, with the lack of information provided in this narrative, it is not possible to estimate power; thus, more information is needed. Answer B may be correct, but without first addressing the question of adequate power, it would be an inappropriate conclusion to draw. Answer C is incorrect because even though the new drug increased HDL concentration more than the other treatment, it is inappropriate to conclude that it is better because, statistically, it is not.
7. **Answer: B**  
The primary end point in this study, the percentage of subjects at or below the target blood pressure, is nominal data. Subjects at target blood pressure (less than 140/90 mm Hg) are defined as having reached the target. This type of data requires either a chi-square test or a Fisher exact test (depending on the sample size or, more accurately, the number of counts in the individual contingency table cells) (Answer B is correct). An independent samples t-test is not appropriate because actual blood pressure values are not being compared (at least not in this question or this end point) (Answer A is incorrect). If we were comparing the actual blood pressure between the two groups, the test might be appropriate if parametric assumptions were met. The Wilcoxon signed-rank test is the appropriate nonparametric test for comparing paired samples (usually in a crossover trial) (Answer C is incorrect). Finally, a one-sample t-test is used to compare the mean of a single group with the mean of a reference group. This is also incorrect in this situation because two groups are being compared (Answer D is incorrect).

8. **Answer: A**  
Detecting the smaller difference between the treatments requires more power. Power can be increased in several different ways. Answer A is correct because the most common approach is to increase the sample size, which is expensive for the researchers. Answer D is incorrect because smaller sample sizes diminish a study’s ability to detect differences between groups. Power can also be increased by increasing \( \alpha \), but doing so increases the chances of a type I error. Answer B decreases \( \alpha \), thus making it more difficult to detect differences between groups. Answer C certainly makes it easier to detect a difference between the two groups, but it uses an unconventional \( \alpha \) value and is thus not the most appropriate technique.

9. **Answer: C**  
Regression analysis is the most effective way to develop models to predict outcomes or variables (Answer C is correct). There are many different types of regression, but all share the ability to evaluate the impact of multiple variables simultaneously on an outcome variable. Correlation analysis is used to assess the association between two (or more) variables, not to make predictions (Answer A is incorrect). Kaplan-Meier curves are used to graphically depict survival curves or time to an event (Answer B is incorrect). Confidence intervals are not used to make predictions (Answer D is incorrect).